Kansas College and Career Ready Standards For Mathematics

Flip Book

Grade 7

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This project used the work done by the Departments of Educations in Ohio, North Carolina, Georgia, engageNY, NCTM, and the Tools for the Common Core Standards.

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About the Flip Books

The development of the "flip books" is in response to the adoption of the Common Core State Standards by the state of Kansas in 2010. Teachers who were beginning the transition to the new Kansas Standards—Kansas College and Career Ready Standards (KCCRS) needed a reliable starting place that contained information and examples related to the new standards.

This project attempts to pull together, in one document some of the most valuable resources that help develop the intent, the understanding and the implementation of the KCCRS. The intent of these documents is to provide a starting point for teachers and administrators to begin unraveling the standard and is by no means the only necessary or complete resource that supports implementation of KCCRS.

This project began in the summer 2012 with the work of Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books". The "flip books" are based on a model that Kansas had for earlier standards however, this edition is far more comprehensive than those in the past. The current editions incorporate the resources from: other state departments of education, documents such as the content progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. The current product was a compilation of work from the project developers in conjunction with many mathematics educators from around the state. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KATM website at www.katm.org and will continue to undergo changes periodically. When significant changes/additions are implemented the necessary modification will be posted and dated.

The initial development of the current update to the "flip books" was driven by the need expressed by teachers of mathematics in Kansas and with the financial support from Kansas Department of Education and encouragement from the Kansas Association of Teachers of Mathematics. These "flip books" have become an ongoing resource that will continue to evolve as more is learned about high quality instruction for the KCCRS for mathematics.

For questions or comments about the flipbooks please contact Melisa Hancock at melisa@ksu.edu.

Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with “grain size”, clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in “Additional and Sample clusters”. Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with “+” within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters’ headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.

DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. See Table 6 Appendix, Depth of Knowledge (DOK)

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 7

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 7 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 7 start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They solve real world problems involving ratios and rates and discuss how they solved them. They see the meaning of a problem and look for efficient ways to represent and solve it. They check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" They understand the approaches of others to solving complex problems and identify correspondences between the different approaches. Example: Seventh graders should navigate among tables, graphs, and equations representing linear relationships to gain insights into the role played by constant rate of change.
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 7 make sense of quantities and their relationships in problem situations. They represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. They *contextualize* to understand the meaning of the number or variable as related to the problem. They *decontextualize* to manipulate symbolic representations by applying properties of operations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. Examples: 1)They apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems, 2) they solve problems involving unit rates by representing the situations in equation form, and 3) they use properties of operation to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 7 understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plot, dot plots, histograms, etc) Example: Use of numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that $5 - 2x$ is equivalent to $3x$. Proficient MS students progress from arguing exclusively through concrete referents such as physical objects and pictorial referents, to also including symbolic representations such as expressions and equations.

4) Model with mathematics.	Mathematically proficient students Grade 7 can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They analyze relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the mode if it has not served its purpose. Examples: Seventh grade students might apply proportional reasoning to plan a school event or analyze a problem in the community, or they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 7 consider the available tools when solving a mathematical problem. These tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software. They are sufficiently familiar with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. Examples: Use graphs to model functions, algebra tiles to see how properties of operations apply to equations, and dynamic geometry software to discover properties of parallelograms. They might use a computer applet demonstrating Archimedes' procedure for approximating the value of π.
6) Attend to precision.	Mathematically proficient students in Grade 7 try to communicate precisely to others. They try to use clear definitions in discussions with others and in their *own* reasoning. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Examples: 1) Seventh grade students can use the definition of rational numbers to explain why a number is irrational, and describe congruence and similarity in terms of transformations in the plane and 2) they accurately apply scientific notation to large numbers and use measures of center to describe data sets.
7) Look for and make use of structure.	Mathematically proficient students in Grade 7 look for and notice patterns and then articulate what they *see*. They can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y.* Examples: 1) Seventh grade students might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see the equation $3x = 2y$ represents a proportional relationship with a unit rate of 3/2 = 1.5, 2) they might recognize how the Pythagorean theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 7 notice if calculations are repeated and look both for general methods and for shortcuts. By paying attention to the calculation of slope as they repeatedly check whether po9ints are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. Examples: 1)By working with tables of equivalent ratios, seventh graders can deduce the corresponding multiplicative relationships and connections to unit rates, 2) they notice the regularity with which interior angle sums increase with the number of sides in a polygon leads to a general formula for the interior angle sum of an *n*-gon, 3) Seventh graders learn to see subtraction as addition of opposite, and use this in a general purpose tool for collecting like terms in linear expressions.

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. **Make sense of problems and persevere in solving them.** • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. **Reason abstractly and quantitatively.** • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is ______related to ______? • What is the relationship between _______and ______? • What does _______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. **Construct viable arguments and critique the reasoning of others.** • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _________? / How could you prove that._____? Will it still work if._____? • What were you considering when.______? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about.______? • How could you demonstrate a counter-example?
4. **Model with mathematics.** • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. **Use appropriate tools strategically.** • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative? • Why was it helpful to use.______? • What can using a _____ show us, that ____may not? • In what situations might it be more informative or helpful to use.______?
6. **Attend to precision.** • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • Is there a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties can you use to explain.____? • How could you test your solution to see if it answers the problem?
7. **Look for and make use of structure.** • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	• What observations do you make about._____? • What do you notice when._____? • What parts of the problem might you eliminate, simplify? • What patterns do you find in._____? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to._______? • In what ways does this problem connect to other mathematical concepts?
8. **Look for and express regularity in repeated reasoning.** • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	• Will the same strategy work in other situations? • Is this always true, sometimes true or never true? • How would we prove that._________? • What do you notice about._________? • What is happening in this situation? • What would happen if.________? • What Is there a mathematical rule for.______? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice?

Critical Areas for Mathematics in 7th Grade

In Grade 7, instructional time should focus on four critical areas: (1)developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

a. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

b. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

c. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three- dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two- dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

d. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Grade 5 Content Standards Overview

Ratios and Proportional Relationships (RP)

- Analyze proportional relationships and use them to solve real-world and mathematical problems**.**

  7.RP.1 7.RP.2 7.RP.3

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

  7.NS.1 7.NS.2 7.NS.3

Expressions and Equations

- Use properties of operations to generate equivalent expressions.

  7.EE.1 7.EE.2
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

  7.EE.3 7.EE.4

Geometry

- Draw construct, and describe geometrical figures and describe the relationships between them.

  7.G.1 7.G.2 7.G.3
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

  7.G.4 7.G.5 7.G.6

Statistics and Probability

- Use random sampling to draw inferences about a population.

  7.SP.1 7.SP.2
- Draw informal comparative inferences about two populations.

  7.SP.3 7.SP.4
- Investigate chance processes and develop, use and evaluate probability models.

  7.SP.5 7.SP.6 7.SP.7 7.SP.8

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Ratios and Proportional Relationships (RP)

Cluster: *Analyze proportional relationships and use them to solve real-world and mathematical problems.*

Standard: Grade 7.RP.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.*

Suggested Standards for Mathematical Practice (MP):

- ✓ *MP.1 Make sense of problems and persevere in solving them.*
- ✓ *MP.2 Reason abstractly and quantitatively.*
- ✓ *MP.3 Construct viable arguments and critique the reasoning of others.*
- ✓ *MP.4 Model with mathematics.*
- ✓ *MP.5 Use appropriate tools strategically.*
- ✓ *MP.6 Attend to precision.*
- ✓ *MP.7 Look for and make use of structure.*
- ✓ *MP.8 Look for and express regularity in repeated reasoning.*
- ✓

Connections:

This cluster is connected to:

- Grade 7 Critical Area of Focus #1: **Developing understanding of and applying proportional relationships** and
- Critical Area of Focus #2**: Developing understanding of operations with rational numbers and working with expressions and linear equations.**
- This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System(Grade 6)
- Relates to Expressions and Equations (Grade 7). Cross curricular connections - economics, personal finance, reading strategies.

Explanations and Examples:

Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. For example, if $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then the amount of paint needed for the entire wall can be computed by $\frac{1}{2}$ gal ÷ by $\frac{1}{6}$ wall $\frac{\frac{1}{2}}{\frac{1}{6}}$. This calculation gives 3 gallons. This standard requires only the use of ratios as fractions. Fractions may be proper or improper.

Instructional Strategies:

Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions.

Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a "rule" that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc.

This is not the time for students to learn to cross multiply to solve problems.

Although algorithms provide efficient means for finding solutions, *the cross-product algorithm* commonly used for solving proportions ***will not*** aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on "out of 100"; now percents above 100 are encountered.

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions.

Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar problem to a given problem and explain their reasoning.

Common Misconceptions:

Students may confuse the significance of the numerator compared to the denominator.

Students may believe that the denominator with a great digit automatically has a greater value than a fraction with a lesser denominator, e.g. , $\frac{1}{8} > \frac{1}{3}$.

Students may rely on one configuration for setting up proportions without realizing that other configurations may also be correct (*within ratios* and *between ratios*).

Students may have difficulty calculating unit rate, recognizing unit rate when it is graphed on a coordinate plane, and realizing that unit rate is also the slope of a line.

Students may misinterpret or not have mastery of the precise meanings and appropriate use of ratio and proportion vocabulary.

Students may miscomprehend the difference between additive reasoning versus multiplicative reasoning.

Tools/Resources:

For detailed information see Ratios and Proportional Reasoning Learning Progression.

- 7.RP Track Practice
- 7.RP Stock Swaps, Variation 2
- 7.RP Stock Swaps, Variation 3
- 7.RP Sale!
- 7.RP Thunder and Lightning
- 6.RP, 7.RP.3 Climbing the steps of El Castillo
- 7.RP Dueling Candidates
- 7.RP Track Practice
- 7.RP Cooking with the Whole Cup
- 7.RP Molly's Run
- 7.RP Molly's Run, Assessment Variation

"See Saw Nickels", Georgia Department of Education. Students focus on extending their conceptual understanding of proportional relationships and direct variation to include inverse relationships. Students will use manipulatives, completed charts, and graphs to further their understanding.

Domain: Ratios and Proportional Relationships (RP)

Cluster: *Analyze proportional relationships and use them to solve real-world and mathematical problems*

Standard: Grade 7.RP.2

Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: **See Grade 7.RP. 1**

Explanations and Examples:

Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade.
Students determine if two quantities are in a proportional relationship from a table.
For example, the table below gives the price for different number of books.
Do the numbers in the table represent a proportional relationship?

Number of Books	Price
1	3
3	9
4	12
7	18

Students can examine the numbers to see that 1 book at 3 dollars is equivalent to 4 books for 12 dollars since both sides of the tables can be multiplied by 4. However, the 7 and 18 are not proportional since 1 book multiplied by 7 and 3 dollars multiplied by 7 will not give 7 books for 18 dollars. Seven books for $18 is not proportional to the other amounts in the table; therefore, there is not a constant of proportionality.

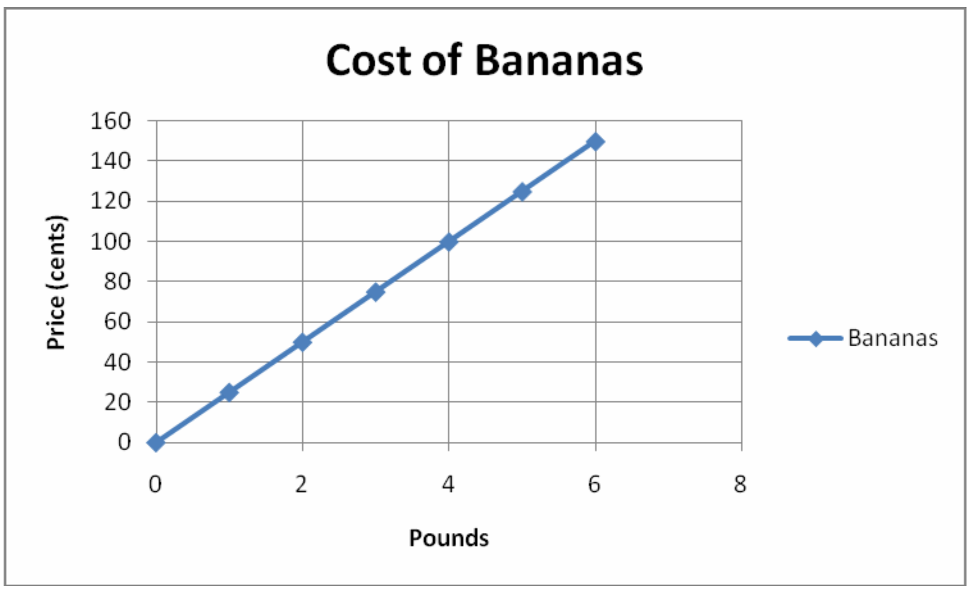
Explanations and Examples:

Students graph relationships to determine if two quantities are in a proportional relationship and interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books cost 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The *y*-coordinate when *x* = 1 will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

The graph below represents the price of the bananas at one store. What is the constant of proportionality? From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph.

Note: Any point on the graph will yield this constant of proportionality.



The cost of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds. What is the constant of proportionality (unit rate)?
Students write equations from context and identify the coefficient as the unit rate which is also the constant of proportionality.

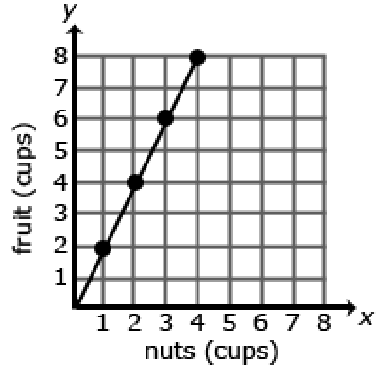
*Note***:** This standard focuses on the representations of proportions. Solving proportions is addressed in **7.SP.3**.

Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a constant of proportionality equal to the slope of the line.

Examples:

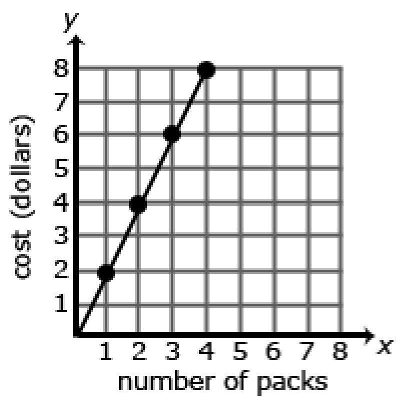
A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
Cups of Nuts (x)	1	2	3	4
Cups of Fruit (y)	2	4	6	8



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1). The constant of proportionality is shown in the first column of the table and by the slope of the line on the graph.

The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2 per pack. Represent the relationship using a table and an equation.



Solution:

Table:

Number of Packs of Gum	**Cost in Dollars**
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where *d* is the cost in dollars and g is the packs of gum.

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as "the number of packs of gum times the cost for each pack is the total cost in dollars". They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table.

The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost $g \times 2 = d$.

Tools/Resources:

For detailed information see Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations – Fractions.

- 7.RP Music Companies, Variation 1
- 7.RP Art Class, Variation 1
- 7.RP Art Class, Variation 2
- 7.RP Buying Coffee
- 7.RP Sore Throats, Variation 1
- 7.RP Robot Races
- 7.RP Robot Races, Assessment Variation
- 7.RP Art Class, Assessment Variation
- 7.RP Buying Bananas, Assessment Version
- 7.RP Walk-a-thon 2
- 7.RP Proportionality

"Walking to Scoops", Georgia Department of Education. Students use a real-world scenario to explore the walking rate on time and distance traveled.

"The Final Challenge", Georgia Department of Education. Students construct plane figures to create a regular octagon using tools and construction techniques.

"Similar Triangles", Georgia Department of Education. Students use an object perpendicular to the ground and measurement tool and their shadow to determine height of objects.

"Feeding Frenzy", Illuminations Lesson-students multiply and divide a recipe to feed groups of various sizes. Students will use unit rates and proportions and think critically about real world applications of a backing problem.

Domain: Ratios and Proportional Relationships (RP)

Cluster: *Analyze proportional relationships and use them to solve real-world and mathematical problems.*

Standard: Grade 7.RP.3

Use proportional relationships to solve multistep ratio and percent problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: **See Grade 7.RP. 1**

Explanations and Examples:

In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication. Students understand the mathematical foundation for cross-multiplication.

For example, a recipe calls for $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk.
If you increase the recipe to use 3 cups of milk, how many teaspoons of butter are needed?
Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

$$\frac{\frac{3}{4}}{2} = \frac{x}{3}$$

The use of proportional relationships is also extended to solve percent problems involving tax, markups and markdowns simple interest ($I = prt, I = interest, p = principle, r = rate, and\ t = time$), gratuities and commissions, fees, percent increase and decrease, and percent error.

For example, Games Unlimited buys video games for $10. The store increases the price 300%? What is the price of the video game?

> Using proportional reasoning, if $10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30 would be $10 times 3. Thirty dollars represents the amount of increase from $10 so the new price of the video game would be $40.

Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error.

$$\% \; error = \frac{|your \; result - accepted \; value|}{accepted \; value} \times 100\%$$

For example, you need to purchase a countertop for your kitchen. You measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is the percent error?

$$\% \; error = \frac{|5ft - 4.5ft|}{4.5} \times 100$$

$$\% \; error = \frac{|0.5ft|}{4.5} \times 100$$

Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

Examples:

Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs $4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say: "The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40."

$$\$4.17 + \$4.17 + (0.24 \cdot 4.17) = 2.24 \times 4.17$$

100%	100%	24%
$4.17	$4.17	?

A sweater is marked down 33%. Its original price was $37.50.
What is the price of the sweater before sales tax?

$37.50 Original Price of Sweater	
33% of $37.50	67% of $37.50 Sale Price of sweater

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

A shirt is on sale for 40% off. The sale price is $12. What was the original price?
What was the amount of the discount?

Discount 40% of Original Price	Sale Price - $12 60% of original price
Original Price (p)	

At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.

A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal?

After eating at a restaurant, your bill before tax is $52.50. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

$$The\ Amount\ Paid = 0.20 \times \$52.50 + 0.08 \times \$52.50 = 0.28 \times \$52.50$$

Instructional Strategies: See **Grade 7.RP.1**

Tools & Resources:

- Illustrative Mathematics: Buying Protein Bars and Magazines
- Illustrative Mathematics: Chess Club
- Illustrative Mathematics: Comparing Years
- Illustrative Mathematics: Friends meeting on bikes
- Illustrative Mathematics: Music Companies, Variation 2
- Illustrative Mathematics: Selling Computers
- Illustrative Mathematics: Tax and Tip
- Illustrative Mathematics: Sand Under the Swing Set
- Illustrative Mathematics: Stock Swaps, Variation 2
- Illustrative Mathematics: Sale!

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: **Grade 7.NS.1**

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
- **b.** Understand $p + q$ as the number located a distance $|q|$ from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$
- **d.** Apply properties of operations as strategies to add and subtract rational numbers. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure

Connections:

This cluster is connected to:

- Grade 7 Critical Area of Focus #2: Developing understanding of operations with rational numbers and working with expressions and linear equations.

Explanations and Examples:

Students add and subtract rational numbers using a number line. For example, to add -5 + 7, students would find -5 on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. *Students also add negative fractions and decimals and interpret solutions in given contexts.*

Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.

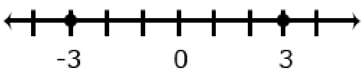
Examples:

Use a number line to illustrate:

$$p - q$$
$$p + (-q)$$

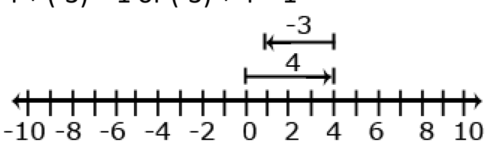
Is this equation true: $p - q = p + (-q)$

-3 and 3 are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and it's opposite is zero.



You have \$4 and you need to pay a friend \$3. What will you have after paying your friend?

4 + (-3) = 1 or (-3) + 4 = 1



Name a number that makes each statement true. Justify your solution by showing a model or providing an explanation.

- $-4.8 + ? = a\ positive\ number$
- $? - \frac{3}{2} = a\ negaitve\ number$
- $-2.15 - ? = a\ negative\ number$

Instructional Strategies:

This cluster (7.NS.1-3) builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using + or – as having opposite directions or values,
- points on a number line show distance and direction,
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself,
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers. Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined.

Number lines present a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board.

Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.

Fractional rational numbers and whole numbers should be used in computations and explorations.

Students should be able to give contextual examples of integer operations, write and solve equations for real- world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers. For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =
4 x - 4 =	-4 x 4 =	-4 x - 4 =

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \times -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and communtativity of multiplication. Then students should be asked to answer these questions and prove their responses:

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language $\left(-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}\right)$ is written for the teacher's information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.
In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

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Resources/Tools:
For detailed information, see Progressions for the Common Core State Standards in Mathematics: Number System 6-8.

- 7.NS Comparing Freezing Points
- 7.NS Operations on the number line
- 7.NS Distances on the Number Line 2
- 7.NS, 7.EE Bookstore Account
- 7.NS Rounding and Subtracting
- 7.NS Distances Between Houses
- 7.NS Differences and Distances
- 7.NS Differences of Integers
- 7.NS Differences of Integers
- 7.NS Differences of Integers

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: **Grade 7.NS.2**

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $-1 \times -1 = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $\left(-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}\right)$. Interpret quotients of rational numbers by describing real-world contexts.
- **c.** Apply properties of operations as strategies to multiply and divide rational numbers.
- **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision

Connections: See **Grade 7.NS.1**

Explanations and Examples:

Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign. Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for work with rational and irrational numbers in 8th grade. For example, identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5.)

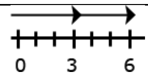
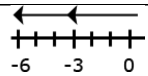
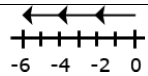
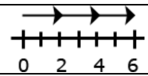
Multiplication and division of integers is an extension of multiplication and division of whole numbers.

Examples

Examine the family of equations. What pattern do you see?

Create a model and context for each of the products.

Write and model the family of equations related to $2 \times 3 = 6$.

Equation	Number Line Model	Context
$2 \times 3 = 6$		Selling two packages of apples at $3.00 per pack
$2 \times -3 = -6$		Spending 3 dollars each on 2 packages of apples
$-2 \times 3 = -6$		Owing 2 dollars to each of your three friends
$-2 \times -3 = 6$		Forgiving 3 debts of $2.00 each

Instructional Strategies:

Instruction needs to focus on developing understanding of operations with rational numbers. Students need multiple opportunities to develop a unified understanding of number, recognizing fractions, decimals (that have a finite or repeating decimal representation), and percent as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g. amounts owed or temperatures below zero), students explain and interpret the rules for "operating" with negative numbers.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Tools/Resources:

7.NS Equivalent fractions approach to non-repeating decimals

7.NS Repeating decimal as approximation

7.NS.2d Decimal Expansions of Fractions

Common Misconceptions:

- Students may incorrectly use integer rules
- Students may confuse the absolute value symbol with the number one
- Students may incorrectly use the additive inverse when working with operations of integers
- Students may have confusion and misapplication of a complex fractions
- Students may think that a number divided by zero is zero rather than undefined

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: Grade 7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Suggested Standards for Mathematical Practice (MP):

MP.1 Make sense of problems and persevere in solving them.

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.NS.2**

Explanations and Examples:

Students use order of operations from 6th grade to write and solve problem with all rational numbers.

Examples:

Your cell phone bill is automatically deducting $32 from your bank account every month.
How much will the deductions total for the year?

$$32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12(-32)$$

It took a submarine 20 seconds to drop to 100 feet below sea level from the surface.
What was the rate of the descent?

$$\frac{-100\ feet}{20\ seconds} = \frac{-5\ feet}{1\ second} = -5\ ft/sec$$

The three seventh grade classes at Ft. Riley Middle School collected the most box tops for a school fundraiser, and so they won a $600 prize to share between them. Mrs. Molt's class collected 3,760 box tops, Mrs. Johnson's class collected 2,301, and Mr. Handlos' class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

A teacher might start out by asking questions like, "Which class should get the most prize money?
Should Mrs. Molt's class get more or less than half of the money?
Mrs. Molt's class collected about twice as many box tops as Mr. Handlos' class - does that mean that
Mrs. Molt's class will get about twice as much prize money as Mr. Canyon's class?"

This task represents an opportunity for students to engage in Standard MP.5 *Use appropriate tools strategically.* There is little benefit in students doing the computations by hand (few adults would), and so provides an opportunity to discuss the value of having a calculator and when it is (and is not) appropriate to use it.

***Sample Solution*:**

All together, the students collected $3{,}750 + 2{,}301 + 1{,}855 = 7{,}916$ box tops.

Mr. Molt's class collected $\frac{3760}{7916}$of the box tops.

The amount for Mrs. Molt's class is $\left(\frac{3760}{7916}\right) 600 \approx 284.99$

Mrs. Johnson's class collected $\frac{2301}{7916}$of the box tops.

The amount for Mrs. Johnson's class is $\left(\frac{2301}{7916}\right) 600 \approx 174.41$

Mr. Handlos' class collected $\frac{1855}{7916}$of the box tops.

The amount for Mr. Handlos' class is $\left(\frac{1855}{7916}\right) 600 \approx 140.60$

$284.99 should go to Mrs. Molt's class, $174.41 should go to Mrs. Johnson's class, and $140.60 should go to Mr. Handlos' class.

Instructional Strategies: See **Grade 7.NS. 1-2**

See Also: Number Systems (Grade 6-8) and Number High School

Illustrative math-"Sharing Prize Money":

For detailed information, see Progressions for the Common Core State Standards in Mathematics: Number System 6-8.

Domain: Expressions and Equations (EE)

Cluster: *Use properties of operations to generate equivalent expressions*

Standard: **Grade 7.EE.1**

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure

Connections: **(7.EE.1-2)**

This cluster is connected to:

- Grade 7 Critical Area of Focus #2: **Developing understanding of operations with rational numbers and working with expressions and linear equations**.
- The concepts in this cluster build from Operations and Algebraic Thinking -write and interpret numerical expressions from Grade 5 and provides the foundation for equation work in Grade 8.
- It also assists in building the foundational work for writing equivalent non-linear expressions in the High School Conceptual Category - Algebra.

Explanations and Examples:

This is a continuation of work from 6^{th} grade using properties of operations and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive / negative fractions and decimals) to write equivalent expressions.

Examples:

Write an equivalent expression for $3(x+5)-2$.

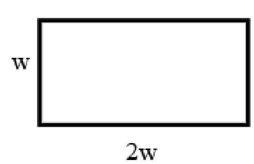
Suzanne thinks the two expressions $2(3a-2)+4a$ and $10a-2$ are equivalent? Is she correct? Explain why or why not?

Write equivalent expressions for: $3a+12$.

Possible solutions might include factoring as in $3(a+4)$, or other expressions such as $a+2a+7+5$.

- A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w+w+2w+2w$. Write the expression in two other ways.

Solution: $6w$ or $2(w)+2(2w)$.



An equilateral triangle has a perimeter of $6x + 15$. What is the length of each of the sides of the triangle?

Solution: $3(2x + 5)$, therefore each side is $2x + 5$ units long.

For numbers 1a–1e, select Yes or No to indicate whether each of these expressions is equivalent to $2(2x + 1)$.

1a.	$4x + 2$	Yes	No
1b.	$2(1 + 2x)$	Yes	No
1c.	$2(2x) + 1$	Yes	No
1d.	$2x + 1 + 2x + 1$	Yes	No
1e.	$x + x + x + x + 1 + 1$	Yes	No

Solution:

1a.	Y - Equivalent by distributive property
1b.	Y - Equivalent by commutative property
1c.	N - Not equivalent by misapplying distributive property
1d.	Y - Equivalent by understanding 2 as a factor
1e.	Y - Equivalent by understanding 2 as a factor and distributive property $2x=(x+x)$

Instructional Strategies:

Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers.

Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example $5(3 + 2x)$ is equal to $5 \bullet 3 + 5 \bullet 2x$ Let $x = 6$ and substitute 6 for x in both equations.

$5(3 + 2\bullet6)$	$5\bullet3 + 5\bullet2\bullet6$
$5(3 + 12)$	$15 + 60$
$5(15)$	75
75	

Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication.

Tools/Resources:

For detailed information, see Learning Progressions for Expressions and Equations:
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

7.EE Miles to Kilometers
7.EE Equivalent Expressions?
7.EE Writing Expressions

Common Misconceptions:

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations.

For example, having a student simplify an expression like $8 + 4(2x - 5) + 3x$ can bring to light several misconceptions.

- Do the students immediately add the 8 and 4 before distributing the 4?
- Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis?
- Do they collect all like terms $8 + 4 – 5$, and $2x + 3x$?

Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.

Domain: Expressions and Equations (EE)

Cluster*: Use properties of operations to generate equivalent expressions.*

Standard: **Grade 7.EE.2**

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 7.EE.1**

Explanations and Examples:

Students understand the reason for rewriting an expression in terms of a contextual situation. For example, students understand that a 20% discount is the same as finding 80% of the cost (.80c).

All varieties of a brand of cookies are $3.50. A person buys 2 peanut butter, 3 sugar and 1 chocolate. Instead of multiplying 2 x $3.50 to get the cost of the peanut butter cookies, 3 x $3.50 to get the cost of the sugar cookies and 1 x $3.50 for the chocolate cookies and then adding those totals together. Students recognize that multiplying $3.50 times 6 will give the same total.

Examples:

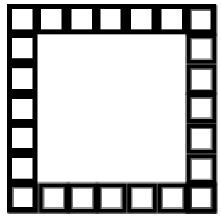
Jamie and Ted both get paid an equal hourly wage of $9 per hour. This week, Ted made an additional $27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way?

Students may create several different expressions depending upon how they group the quantities in the problem.
Possible student responses:

- To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the $27 overtime to find the total wages for the week. The student would write the expression $9J + 9T + 27$.
- To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$
- To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the $27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$

Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent.

Which expression do you think is most useful? Explain your thinking.



Instructional Strategies: 7.EE.3; (See **7.EE.1**)

Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: $b - 0.10b = 1.00b - 0.10b$ which equals $0.90b$ or 90% of the amount.

Resources/Tools:

7.EE Ticket to Ride

Common Misconceptions: See **Grade 7.EE.1**

Domain: Expressions and Equations (EE)

***Cluster**: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*

Standard: Grade 7.EE.3

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Grade 7 Critical Area of Focus #2: **Developing understanding of operations with rational numbers and working with expressions and linear equations**
- Critical Area of Focus #3: **solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.

Explanations and Examples:

Students solve contextual problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers.

Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations.

Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),

- using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (student's select close whole numbers for fractions or decimals to determine an estimate).

Examples:

The youth group is going on a trip to the state fair. The trip costs \$52. Included in that price is \$11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths.

Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

x	x	11
52		

$$2x + 11 = 52$$
$$2x = 41$$
$$x = \$20.50$$

Renee, Susan, and Martha will share the cost to rent a vacation house for a week.

- Renee will pay 40% of the cost.
- Susan will pay 0.35 of the cost.
- Martha will pay the remainder of the cost.

Part A

Martha thinks she will pay $^1/_3$ of the cost. Is Martha correct?
Use mathematics to justify your answer.

Part B

The cost to rent a vacation house for a week is \$850. How much will Renee, Susan, and Martha each pay to rent this house for a week?

Sample Response:

Part A: Martha is incorrect. She will pay $\frac{\mathbf{1}}{\mathbf{4}}$ of the cost.

$$1 - (40\% + 0.35)$$
$$1 - (0.40 + 0.35)$$
$$1 - 0.75 = 0.25 = \frac{25}{100} = \frac{1}{4}$$

Part B

Renee: $0.40 \times \$850 = \$340$

Susan: $0.35 \times \$850 = \$297.50$

Martha: $0.25 \times \$850 = \$212.50$

When working on a report for class, Catrina read that a woman over the age of 40 can lose approximately 0.06 centimeters of height per year.

a. Catrina's aunt Nancy is 40 years old and is 5 feet 7 inches tall. Assuming her height decreases at this rate after the age of 40, about how tall will she be at age 65? (Remember that 1 inch = 2.54 centimeters.)

b. Catrina's 90-year-old grandmother is 5 feet 1 inch tall. Assuming her grandmother's height has also decreased at this rate, about how tall was she at age 40? Explain your reasoning.

Solution:
Convert the rate of shrinkage to inches per year.
Note that there is a significant amount of rounding in the final answer. This is because people almost never report their heights more precisely than the closest half-inch. If we assume that the heights reported in the task stem are rounded to the nearest half-inch, then we should report the heights given in the solution at the same level of precision.

If a person loses an average of 0.06 cm per year after age 40 and $1\ inch = 2.54\ cm$, after age 40 they lose, on average $0.06 \div 2.54 = 0.024\ inches\ per\ year$.

a. In the 25 years from age 40 to age 65, Nancy could be expected to lose approximately$25 \times 0.024 = 0.6\ inches$. Subtracting this from Nancy's current height, Nancy's height at age 65 could be expected to be approximately $5\ feet, 6\frac{1}{2}\ inches$.
b. In the 55 years from age 40 to age 90, Catrina's grandmother could be expected to lose approximately twice Nancy's loss in height, or 1.2 inches. Adding this to Catrina's grandmother's current height, Catrina's grandmother could be expected to have been approximately 5 *feet,* 2 *inches* tall at age 40.

Instructional Strategies:
To assist students' assessment of the reasonableness of answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. Connections between performing the inverse operation and undoing the operations are appropriate here. It is appropriate to expect students to show the steps in their work. Students should be able to explain their thinking using the correct terminology for the properties and operations.

Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost.
$C = (12.50 - 0.75)(1 + T) + 0.75 = 11.75(1 + T) + 0.75$ where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Tools/ Resources:

7.EE Shrinking
7.EE Discounted Books
7.RP,EE Gotham City Taxis
6.EE,RP 7.EE,RP Anna in D.C.
7.EE Who is the better batter?
7.G Stained Glass

Common Misconceptions:

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations.

For example having a student simplify an expression like $8 + 4(2x - 5)$ can bring to light several misconceptions.

- Do the students immediately add the 8 and 4 before distributing the 4?
- Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis?
- Do they collect all like terms $8 + 4 - 5$, and $2x + 3x$?

Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.

Domain: Expressions and Equations (EE)

Cluster: *Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*

Standard: Grade 7.EE.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- ***a.*** *Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
- ***b.*** *Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.EE.3**

Explanations and Examples:

Students solve multi-step equations and inequalities derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution. Students graph inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context.

Examples:

Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left.
How much did each pen cost?

The sum of three consecutive even numbers is 48. What is the smallest of these numbers?

Solve: $\frac{5}{4}n + 5 = 20$

Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.

Steven has $25 dollars. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy?

Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution.

- Solve $\frac{1}{2}x+3>2$ and graph your solution on a number line.

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs. of gear for the boat plus 10 lbs. of gear for each person.

a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.

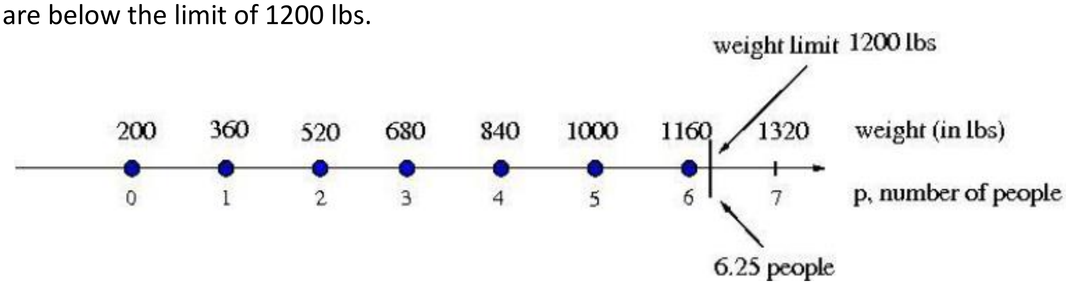
b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

Sample Solution:

a. Let p be the number of people in a group that wishes to rent a boat. Then $150p$ represents the total weight of the people in the boat, in pounds. Also, $10p$ represents the weight of the gear that is needed for each person on the boat. So the total weight in the boat that is contributed solely by the people is $150p + 10p = 160p$

Because each group requires 200 pounds of gear regardless of how many people there are, we add this to the above amount. We also know that the total weight cannot exceed 1200 pounds.
So we arrive at the following inequality: $160p + 200 < 1200$

One possible graph illustrating the solutions is shown below. We observe that our solutions are values of p listed below the number line and shown by the dots, so that the corresponding weights $160p+200$, listed above the line, are below the limit of 1200 lbs.



b. We can find out which of the groups, if any, can safely rent a boat by substituting the number of people in each group for p in our inequality. We see that:
   - For Group 1: 160(4) + 200 = 840 < 1200
   - For Group 2: 160(5) + 200 = 1000 < 1200
   - For Group 3: 160(8) + 200 = 1480 < 1200

We find that both Group 1 and Group 2 can safely rent a boat, but that Group 3 exceeds the weight limit, and so cannot rent a boat. To find the maximum number of people that may rent a boat, we solve our inequality for *p*.

$$160p + 200 < 1200$$
$$160p < 100$$
$$p < 6.25$$

As we cannot have 0.25 person, we see that 6 is the largest number of people that may rent a boat at once. This also matches our graph; since only integer values of *p* make sense, 6 is the largest value of *p* whose corresponding weight value lies below the limit of 1200 lbs.

Instructional Strategies:
Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost.
$C = (12.50 - 0.75)(1 + T) + 0.75 = 11.75(1 + T) + 0.75$ where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Tools/ Resources:
See engageNY Modules:
https://www.engageny.org/resource/grade-7-mathematics

7.EE Fishing Adventures 2
7.RP,EE Gotham City Taxis
7.NS, 7.EE Bookstore Account
7.EE Sports Equipment Set

Common Misconceptions: See **Grade 7.EE.3**

Domain: Geometry (G)

Cluster: *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: **Grade 7.G.1**

Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Grade 7 Critical Area of Focus #3: **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.
- Connections should be made between this cluster and the Grade 7 Geometry Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (Grade 7.G.4-6)
- Grade 7 Ratios and Proportional Relationships.

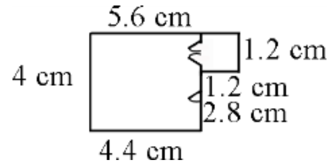
This cluster leads to the development of the triangle congruence criteria in Grade 8.

Explanations and Examples:

Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

Examples:

Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft., what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



Major Supporting Additional Depth Opportunities(DO)

A company designed two rectangular maps of the same region. These maps are described below.

- *Map 1*: The dimensions are 8 inches by 10 inches. The scale is ¾ mile to 1 inch.
- *Map 2*: The dimensions are 4 inches by 5 inches.

Write a ratio that represents the scale on Map 2.

Solution:

$$\frac{3}{4}\ mile\ to\ \frac{1}{2}\ inch$$

Instructional Strategies:

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. For example, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	2.5 inches		
Parallelogram	3.25 centimeters		
Trapezoid	(Actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample recording sheet is shown.

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in.	2	10 inches	20 inches	2	6 sq. in.	24 sq. in.	4
Triangle	1.5 inches	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq. in.	4

Students should move on to drawing scaled figures on grid paper with proper figure labels, scale and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter?

What is the new area? or If the scale is 6, what will the new side length look like? or Suppose the area of one triangle is 16 sq. units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?

Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles.

Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles.

For example, subdividing a polygon into triangles using a vertex $(N-2)180°$ or subdividing polygons into triangles using an interior point $180°N - 360°$ where N = the number of sides in the polygon. An extension would be to realize that the two equations are equal.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube" or "What three-dimensional figure can produce a hexagon slice?" This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Tools/Resources:
See engageNY Modules:
https://www.engageny.org/resource/grade-7-mathematics

7.G Floor Plan
7.G Map distance

Common Misconceptions:
Students often confuse the vocabulary associated with this domain. Teachers should provide experiences for the explicit discovery of these terms to apply meaning through written, pictorial, and experimental means. They continue to misuse units for distance, area, and volume. This too should be explicitly reviewed from the sixth grade domain.

Student's may have misconceptions about correctly setting up proportions, how to read a ruler, doubling side measures and does not double perimeter.

Domain: Geometry (G)

Cluster: *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: **Grade 7.G.2**

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP. 7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.G.1**

Explanations and Examples:

Students understand the characteristics of angles that create triangles. For example, can a triangle have more than one obtuse angle? Will three sides of any length create a triangle? Students recognize that the sum of the two smaller sides must be larger than the third side.

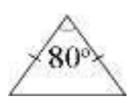
Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity.

Examples:

Is it possible to draw a triangle with a 90˚ angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?

Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not?

Draw an isosceles triangle with only one 80 degree angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?



Can you draw a triangle with sides that are 13 cm, 5 cm and 6 cm?

Draw a quadrilateral with one set of parallel sides and no right angles.

Instructional Strategies:

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles.

Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles.

For example, subdividing a polygon into triangles using a vertex $(N-2)180°$ or subdividing polygons into triangles using an interior point $180°N - 360°$ where N = the number of sides in the polygon. An extension would be to realize that the two equations are equal.

Resources/Tools:

See EngageNY Modules: https://www.engageny.org/resource/grade-7-mathematics

Domain: Geometry (G)

Cluster: *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: **Grade 7.G.3**

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.G.1**

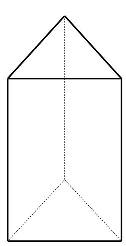
Explanations and Examples:

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right rectangular pyramids, parallel, and perpendicular.**

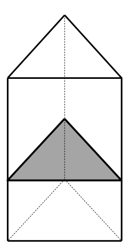
Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and pyramids. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram; cuts made at an angle through the right rectangular pyramid will also produce a parallelogram.

Right Prisms

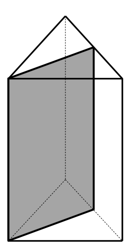
Let us consider the following right prism and then draw some general conclusions.



Initial Diagram



Cross Section Parallel to the Bases



Cross Section Perpendicular to the Bases

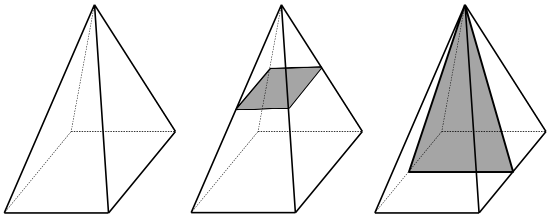
We make the following observations which can be generalized to other right prisms:

1. In the cross section parallel to the bases, the cross section is congruent to the bases.
2. In the cross section perpendicular to the bases, the cross section is a rectangle.

Major | Supporting | Additional | Depth Opportunities(DO)

Right Pyramids

Let us consider the following right pyramid and then draw some general conclusions.



Initial Diagram

Cross Section Parallel to the Bases

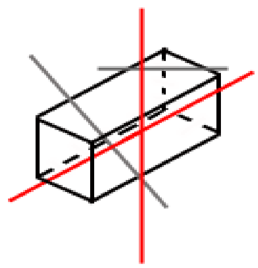
Cross Section Perpendicular to the Bases

We make the following observations which can be generalized to other right pyramids:

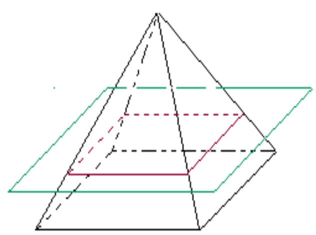
1. In the cross section parallel to the base, the cross section is similar to the base. (In this particular diagram, the base is a square, and any cross section parallel to the base will result in a smaller square.).
2. In the cross section perpendicular to the base, the cross section is a triangle.

Examples:

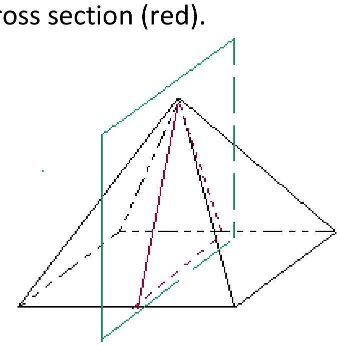
Using a clay model of a rectangular prism, describe the shapes that are created when planar cuts are made diagonally, perpendicularly, and parallel to the base.



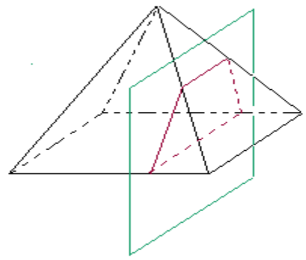
If the pyramid is cut with a plane (green) parallel to the base, the intersection of the pyramid and the plane is a square cross section (red).



If the pyramid is cut with a plane (green) passing through the op vertex and perpendicular to the base, the intersection of the pyramid and the plane is a triangular cross section (red).

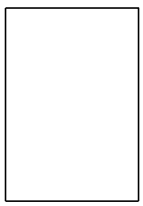


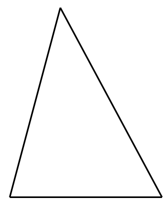
If the pyramid is cut with a plane (green) perpendicular to the base, but not through the top vertex, the intersection of the pyramid and the plane is a trapezoidal cross section (red).
http://intermath.coe.uga.edu/dictnary/descript.asp?termID=95



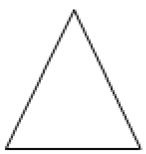
For each of the figures below, sketch a solid which could have the given cross sections.

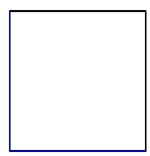
1. Cross section parallel to a base: Cross section perpendicular to a base:





2. Cross section parallel to a base: Cross section perpendicular to a base:





Instructional Strategies:
This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometry problems.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found.

Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube” or “What three-dimensional figure can produce a hexagon slice?” This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Resources/Tools:
7.G Cube Ninjas!

Domain: Geometry (G)

Cluster: *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*

Standard: **Grade 7.G.4**

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

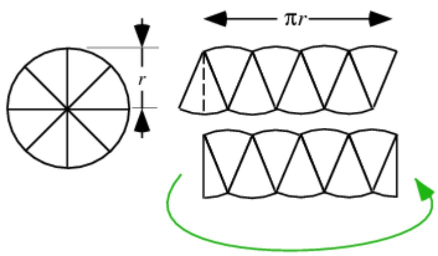
This cluster is connected to:

- Grade 7 Critical Area of Focus #3: **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.
- This cluster builds from understandings of Geometry and in Measurement and Data Grades 3-6.
- It also utilizes the scope of the number system experienced thus far and begins the formal use of equations, formulas and variables in representing and solving mathematical situations.

Explanations and Examples:

Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as Pi. Building on these understandings, students generate the formulas for circumference and area.

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown below, a parallelogram results. Half of an end wedge can be moved to the other end a parallelogram results. The height of the parallelogram is the same as the radius of the circle. The base length is $\frac{1}{2}$ the circumference$(2\pi r)$. The area of the parallelogram (and therefore the circle) is found by the following calculations:
http://mathworld.wolfram.com/Circle.html



$$Area\ of\ Parallelogram = Base \times Height$$
$$Area = \frac{1}{2}(2\pi r) \times r$$
$$Area = \pi r^2$$

Major | Supporting | Additional | **Depth Opportunities(DO)**

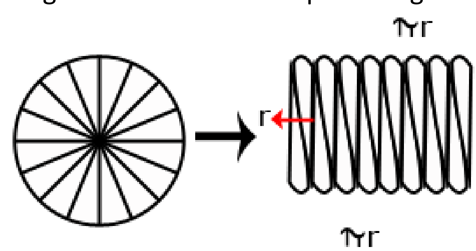
Explanations and Examples:
Students solve problems (mathematical and real-world) including finding the area of left-over materials when circles are cut from squares and triangles or from cutting squares and triangles from circles. "Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area and circumference) and the figure. This should be an expectation for ALL students.

Examples:
The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?

Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures.

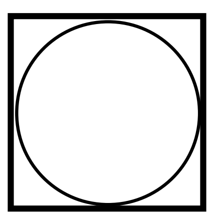
Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles.

Students will use a circle as a model to make several equal parts as you would in a pie model. The greater number the cuts, the better. The pie pieces are laid out to form a shape similar to a parallelogram. Students will then write an expression for the area of the parallelogram related to the radius (note: the length of the base of the parallelogram is half the circumference, or πr, and the height is r, resulting in an area of πr^2.



If students are given the circumference of a circle, could they write a formula to determine the circle's area or given the area of a circle, could they write the formula for the circumference?

An artist used silver wire to make a square that has a perimeter of 40 inches. She then used copper wire to make the largest circle that could fin in the square, as shown below.



How many more inches of silver wire did the artist use compared to cooper wire? (Use π = 3.14)
Show all work necessary to justify your response.

Sample Response:
Each side of the square has a length of $40 \times \frac{1}{4} = 10$ inches.
The radius of the circle is $\frac{10}{2} = 5$ inches, so the circumference of the circle is $2 \times \pi \times 5 = 10 \times 3.14 = 31.4\ inches$.
The perimeter of the square minus the circumference of the circle is $40 - 31.4 = 8.6\ inches$.

Instructional Strategies:

This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi and area.

Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius.

Given multiple-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi and ultimately deriving a formula for circumference. String or yarn laid over the circle and compared to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.

Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Resources/Tools:

7.G The Circumference of a Circle and the Area of the Region it Encloses
7.G Drinking the Lake
7.G Eight Circles
7.G,RP Measuring the area of a circle
7.G Designs
7.G Stained Glass

Illuminations Lesson, "The Ratio of Circumference to Diameter'

Illuminations Lesson, "Geometry of Circles"

Illuminations Lesson, "Discovering the Area Formula for Circles"

Illuminations Lesson, "The Giant Cookie Dilemma"

Illuminations Lesson, "Tetrahedral Kites"

Illuminations Lesson, Popcorn, Anyone?

Illuminations Lesson, Area Contractor

Illustrative Mathematics: "Measuring the Area of a Circle"

Common Misconceptions:
Students may believe that *Pi* is an exact number rather than understanding that 3.14 is just an approximation of *pi*. Many students are confused when dealing with circumference (linear measurement) and area.

This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

Domain: Geometry (G)

Cluster: *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*

Standard: Grade 7.G.5

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.G.4**

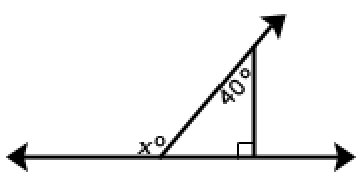
Explanations and Examples:

Students use understandings of angles to write and solve equations.

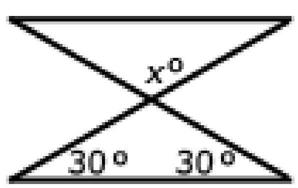
Angle relationships that can be explored include but are not limited to:
Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Examples:

Write and solve an equation to find the measure of angle *x.*



Write and solve an equation to find the measure of angle *x.*



Determine if each of the following statements is always true, sometimes true, or never true.

1. The sum of the measures of two complementary angles is 90°.
2. Vertical angles are also adjacent angles.
3. Two adjacent angles are complementary.
4. If the measure of an angle is represented by x, then the measure of its supplement is represented by $180 - x$.
5. If two lines intersect, each pair of vertical angles are supplementary.

For each statement you chose as "sometimes true," provide one example of when the statement is true and one example of when the statement is not true. Your examples should be a diagram with the angle measurements labeled. If you did not choose any statement as "sometimes true," write "none".

Sample Response:
1: Always true
2: Never true
3: Sometimes true
4: Always true
5: Sometimes true

<u>Statement 3</u>:
Example of True - (two adjacent angles that have a sum of 90°)
Example of Not true - (two adjacent angles that have a sum of 80°)

<u>Statement 5</u>:
Example of True - (two intersecting lines with all angle measurements of 90°)
Example of Not true - (two lines that intersect with no right angles)

Instructional Strategies:

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Resources/Tools:

See engageNY Modules: https://www.engageny.org/resource/grade-7-mathematics

Domain: Geometry (G)

Cluster: *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*

Standard: Grade 7.G.6

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.G.4**

Explanations and Examples:

Students continue work from 5th and 6th grade to work with area, volume and surface area of two- dimensional and three-dimensional objects (composite shapes). Students will not work with cylinders, as circles are not polygons.

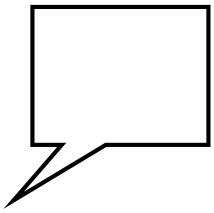
"Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area and volume) and the figure.

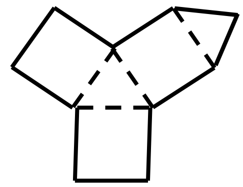
Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area.

Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations.

Examples:

Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?

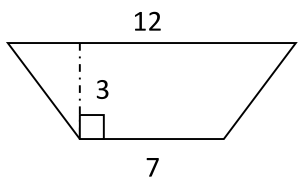




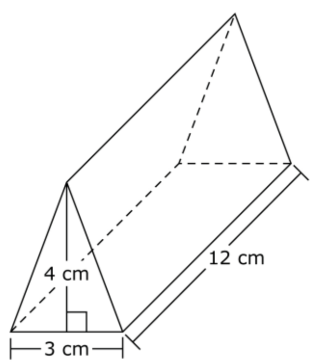
A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.)

Find the area of a triangle with a base length of three units and a height of four units.

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.

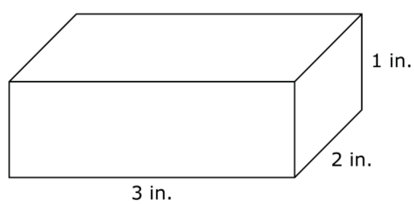


Look at the triangular prism below. Each triangular face of the prism has a base of 3 centimeters (cm) and a height of 4 cm. The length of the prism is 12 cm.



What is the volume of this triangular prism?

Using the rectangular prism shown below, create a new prism with a **surface area** of between 44 square inches and 54 square inches.



Solution: Four prisms should be stacked vertically.

Major Supporting Additional **Depth Opportunities(DO)**

Instructional Strategies:

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Tools/Resources:

7.RP and 7.G Sand Under the Swing Set

Illuminations Lesson, "Popcorn Anyone?"

Illuminations Lesson, "Area Contractor"

Illustrative Mathematics: "Sand Under the Swing Set"

Domain: Statistics and Probability (SP)

Cluster: *Use random sampling to draw inferences about a population.*

Standard: Grade 7.SP.1

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.6 Attend to precision.

Connections:

This cluster is connected to:

- Grade 7 Critical Area of Focus #4: **Drawing inferences about populations based on samples**.
- Initial understanding of statistics, specifically variability and the measures of center and spread begins in Grade 6.

Explanations and Examples:

Students recognize that it is difficult to gather statistics on an entire population. Instead a random sample can be representative of the total population and will generate valid results. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.

Examples:

The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why?

- *Method 1*: Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey.
- *Method 2*: Survey the first 20 students that enter the lunch room.

Amanda asked a random sampling of 40 students from her school to identify their birth month. There are 300 students in her school. Amanda's data is shown in this table.

Students Birth Months

Birth Month	Number of Students
January	3
February	0
March	3
April	10
May	4
June	3
July	4
August	3
September	2
October	2
November	3
December	3

Which of these statements is supported by the data?

- Exactly 25% of the students in Amanda's school have April as their birth month.
- There are no students in Amanda's school that have a February birth month.
- There are probably more students at Amanda's school with an April birth month than a July birth month.
- There are probably more students at Amanda's school with a July birth month than a June birth month.

Instructional Strategies:

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Providing opportunities for students to use real-life situations from science and social studies shows the purpose for using random sampling to make inferences about a population.

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample.

Have students compare the random sample to population, asking questions like "Are all the elements of the entire population represented in the sample?" and "Are the elements represented proportionally?" Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.

Provide students with random samples from a population, including the statistical measures.

Ask students guiding questions to help them make inferences from the sample.

Tools/Resources:

For detailed information, see Progressions for Common Core State Standards in Mathematics: 6-8 Statistics and Probability

7.SP Estimating the Mean State Area
7.SP Election Poll, Variation 2
7.SP Election Poll, Variation 3
7.SP Election Poll, Variation 1

7-SP Mr. Briggs's Class Likes Math

"The Eyes Have It", Georgia Department of Education. Students analyze data and draw conclusions about the data using box-and-whisker plots. Students collect data from experiments on eye blinks.

"See Saw Nickels", Georgia Department of Education. Students focus on extending their conceptual understanding of proportional relationships and direct variation to include inverse relationships. Students will use manipulatives, completed charts, and graphs to further their understanding.

Common Misconceptions:

Students may believe:

One random sample is not representative of the entire population and that many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception. Students' understanding that the random sample must be representative of the population is key to supporting valid inferences.

Domain: Statistics and Probability (SP)

Cluster: *Use random sampling to draw inferences about a population.*

Standard: Grade 7.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 7.SP.1**

Explanations and Examples:

Students collect and use multiple samples of data to answer question(s) about a population. Issues of variation in the samples should be addressed.

Example:

Below is the data collected from two random samples of 100 students regarding student's school lunch preference. Make at least two inferences based on the results.

Lunch Preferences				
Student Sample	**Hamburgers**	**Tacos**	**Pizza**	**Total**
#1	12	14	74	100
#2	12	11	77	100

Instructional Strategies:

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample.

Have students compare the random sample to population, asking questions like "Are all the elements of the entire population represented in the sample?" and "Are the elements represented proportionally?" Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis. Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Resources/Tools:

7.SP.2 – Valentine Marbles

Common Misconceptions: See **Grade 7.SP.1**

Domain: Statistics and Probability (SP)

Cluster: *Draw informal comparative inferences about two populations.*

Standard: Grade 7.SP.3

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This Cluster is connected to:

- Grade 7 Critical Area of Focus #4**: drawing inferences about populations based on samples**. It expands standards 1 and 2 to make inferences between populations.
- Measures of center and variability are developed in Statistics and Probability Grade 6.

Explanations and Examples:

This is the students' first experience with comparing two data sets. Students build on their understanding of graphs, mean, median, Mean Absolute Deviation (M.A.D.) and interquartile range from 6th grade.

Students understand that:

1. a full understanding of the data requires consideration of the measures of variability as well as mean or median,
2. variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap, and
3. median is paired with the interquartile range and mean is paired with the mean absolute deviation.

Students can readily find data as described in the example on sports team or college websites.

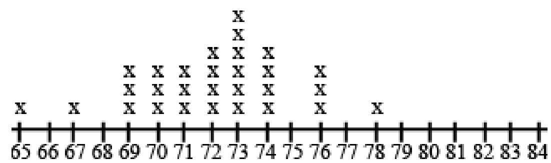
Other sources for data include American Fact Finder (Census Bureau), Fed Stats, Ecology Explorers, USGS, or CIA World Factbook. Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

Example:
Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn't know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists.

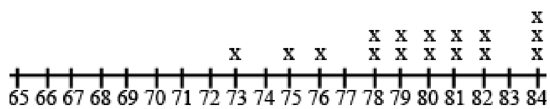
Basketball Team – Height of Players in inches for 2010-2011 Season:
75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84

Soccer Team – Height of Players in inches for 2010-2011 Season:
73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

To compare the data sets, Jason creates a two dot plots on the same scale. The shortest player is 65 inches and the tallest players are 84 inches.



Height of Soccer Players (in)



Height of Basketball Players (in)

In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation to compare the data sets. Jason sets up a table for each data set to help him with the calculations. (*See next page*)

The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches.

The mean absolute deviation (MAD) is calculated by taking the mean of the absolute deviations for each data point. The difference between each data point and the mean is recorded in the second column of the table. Jason used rounded values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set.

The mean absolute deviation is 2.14 inches for the basketball players and 2.53 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams is approximately 3 times the variability of the data sets (7.68 ÷ 2.53 = 3.04).

Soccer Players (n = 29)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
65	-7	7
67	-5	5
69	-3	3
69	-3	3
69	-3	3
70	-2	2
70	-2	2
70	-2	2
71	-1	1
71	-1	1
71	-1	1
72	0	0
72	0	0
72	0	0
72	0	0
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
74	+2	2
74	+2	2
74	+2	2
74	+2	2
76	+4	4
76	+4	4
76	+4	4
78	+6	6
Σ = 2090		**Σ = 62**

Mean = 2090 ÷ 29 =72 inches

MAD = 62 ÷ 29 = 2.13 inches

Basketball Players (n = 16)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
73	-7	7
75	-5	5
76	-4	4
78	-2	2
78	-2	2
79	-1	1
79	-1	1
80	0	0
80	0	0
81	1	1
81	1	1
82	2	2
82	2	2
84	4	4
84	4	4
84	4	4
Σ = 1276		**Σ = 40**

Mean = 1276 ÷ 16 =80 inches

MAD = 40 ÷ 16 = 2.5 inches

Instructional Strategies:

In Grade 6, students used measures of center and variability to describe sets of data. In the cluster "Use random sampling to draw inferences about a population" of Statistics and Probability in Grade 7, students learn to draw inferences about one population from a random sampling of that population.

Students continue using these skills to draw informal comparative inferences about two populations. Provide opportunities for students to deal with small populations, determining measures of center and variability for each population. Then have students compare those measures and make inferences.

The use of graphical representations of the same data (Grade 6) provides another method for making comparisons. Students begin to develop understanding of the benefits of each method by analyzing data with both methods.

When students study large populations, random sampling is used as a basis for the population inference. This builds on the skill developed in the Grade 7 cluster "Use random sampling to draw inferences about a population" of Statistics and Probability.

Measures of center and variability are used to make inferences on each of the general populations.

Then students can make comparisons for the two populations based on those inferences.

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data. Have students investigate how advertising agencies uses data to persuade customers to use their products. Additionally, provide students with two populations and have them use the data to persuade both sides of an argument.

Tools/Resources:
"The Eyes Have It", Georgia Department of Education. Students analyze data and draw conclusions about the data using box-and-whisker plots. Students collect data from experiments on eye blinks.

7.SP.3,4 – Offensive Linemen
7.SP.3,4 – College Athletes

Domain: Statistics and Probability (SP)

Cluster: *Draw informal comparative inferences about two populations.*

Standard: Grade 7.SP.4

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See **Grade 7.SP.3**

Explanations and Examples:

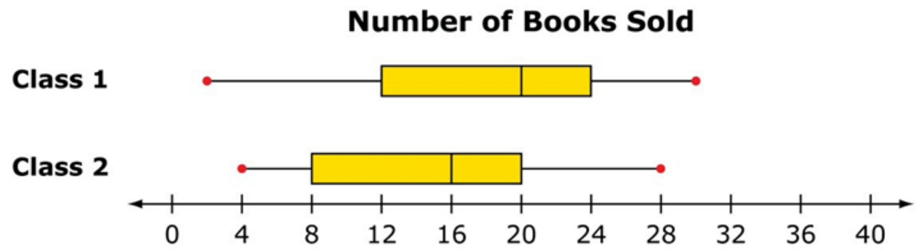
Students are expected to compare two sets of data using measures of center and variability. Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.

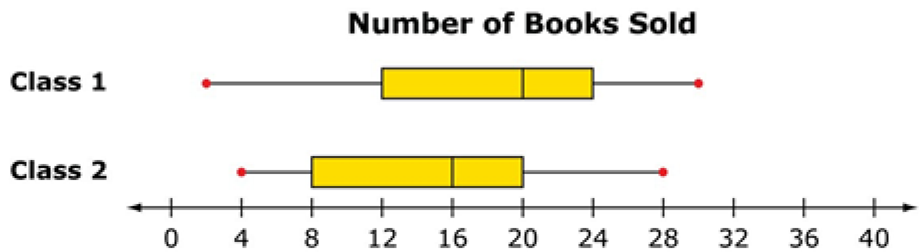
Examples:

The two data sets below depict random samples of the housing prices sold in the King River and Toby Ranch areas of Arizona. Based on the prices below, which measure of center will provide the most accurate estimation of housing prices in Arizona? Explain your reasoning.

- King River: {1.2 million; 242,000; 265,500; 140,000; 281,000; 265,000; 211,000}
- Toby Ranch: {5 million; 154,000; 250,000; 250,000; 200,000; 160,000; 190,000}

The number of books sold by each student in two classes for a fundraiser is summarized by these box plots.





The principal concluded that there was more variability in the number of books sold by Class 1 than Class 2.

Which statement is true about the principal's conclusion? Explain your reasoning.

1. It is valid because the median for Class 1 is greater than the median for Class 2.
2. It is valid because the range for Class 1 is greater than the range for Class 2.
3. It is invalid because the minimum value for Class 1 is less than the minimum value for Class 2.
4. It is invalid because the interquartile range for Class 1 is less than the interquartile range for Class 2.

Sample Response:

1. Not true - statement assumed the median is a measure of variability
2. Correct- supports the principal's statement of more variability as shown by a greater range.
3. Not true- statement assumed the minimum value is a measure of variability
4. Not true – statement did not correctly determine interquartile range

Instructional Strategies: See **Grade 7.SP.3**

Tools/Resources:

See 7.SP.3

Domain: Statistics and Probability (SP)

Cluster: *Investigate chance processes and develop, use, and evaluate probability models.*

Standard: Grade 7.SP.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6.Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections:

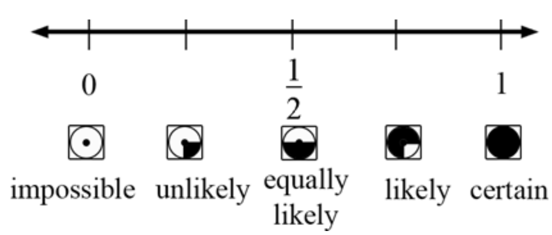
This cluster is connected to:

- This cluster goes beyond the Grade 7 Critical Areas of Focus to address Investigating chance.
- Ratio and Proportional Relationships in Grade 6 is the development of fractions as ratios and percents as ratios. In Grade 7, students write the same number represented as a fraction, decimal or percent.
- Random sampling and simulation are closely connected in Grade 7.SP. Random sampling and simulation is used to determine the experimental probability of event occurring in a population or to describe a population.

Explanations and Examples:

This is students' first formal introduction to probability. Students recognize that all probabilities are between 0 and 1, inclusive, the sum of all possible outcomes is 1. For example, there are three choices of jellybeans – grape, cherry and orange. If the probability of getting a grape is $^{3}/_{10}$ and the probability of getting cherry is $^{1}/_{5}$, what is the probability of getting oranges? The probability of any single event can be recognized as a fraction. The closer the fraction is to 1, the greater the probability the event will occur. Larger numbers indicate greater likelihood. For example, if you have 10 oranges and 3 apples, you have a greater likelihood of getting an orange.

Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line.



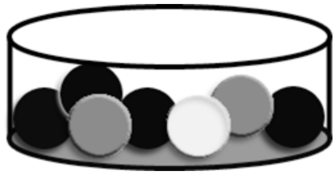
Students can use simulations such as Marble Mania or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

Marble Mania
Random Drawing Tool

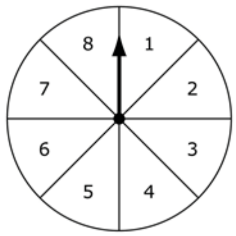
Major Supporting Additional **Depth Opportunities(DO)**

Examples:

A container contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.



Carl and Beneta are playing a game using this spinner.



Carl will win the game on his next spin if the arrow lands on a section labeled 6, 7, or 8. Carl claims it is likely, but not certain, that he will win the game on his next spin.

Explain why Carl's claim is **not** correct.

Beneta will win the game on her next spin if the result of the spin satisfies event *X*. Beneta claims it is likely, but not certain, that she will win the game on her next spin.

Describe an event *X* for which Beneta's claim is correct.

Sample Response:

Carl's claim is incorrect. The probability that Carl will spin a 6 or higher is 0.375. This means that it is more likely that Carl will spin a number less than 6 on his next turn.

For Beneta, event *X* could be "the arrow lands on a section labeled with a number greater than 2."

Instructional Strategies:

Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and $\frac{1}{2}$. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as $\frac{1}{2}$.

Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.

Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome.

Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability. Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities

Common Misconceptions:

Students may attempt to give probability as a number greater than one rather than representing it as a number between zero and one. For example, if there are 2 blue marbles and 3 red marbles, the probability of picking a blue marble is $\frac{2}{5}$, not 2.

Students often expect the theoretical and experimental probabilities of the same data to match.

By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities.

Note examples in simulations when some possibilities are not shown.

Domain: Statistics and Probability (SP)

Cluster: *Investigate chance processes and develop, use, and evaluate probability models.*

Standard: Grade 7.SP.6

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.

Connections: See **Grade 7.SP.5**

Explanations and Examples:

Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability. The focus of this standard is relative frequency -- The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful events.

Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.

Examples:

Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities

How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?

Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. An example would be 3 green marbles, 6 blue marbles, and 3 blue marbles.

Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.

Instructional Strategies: See **Grade 7.SP.5**

Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Tools /Resources:

7.SP Rolling Dice
7.SP Tossing Cylinders
7.SP.6 Heads or Tails

"Odd and Even", Great Tasks for Mathematics Grades 6-12, NCSM, (2013). Students explore experimental probabilities and calculate theoretical probabilities of odd and even sums of random numbers.

Common Misconceptions:

Students may have trouble understanding the difference between the probability that should happen in theory and the outcomes of an actual event. Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students may confuse finding the probability of event A or event B occurring (either one could occur) vs. the probability of even A and even B both occurring (compound even).

Domain: Statistics and Probability (SP)

Cluster: *Investigate chance processes and develop, use, and evaluate probability models.*

Standard: Grade 7.SP.7

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning

Connections: See **Grade 7.SP.5**

Explanations and Examples:

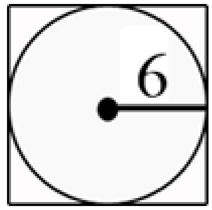
Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data.

Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web- based simulations. Students can also develop models for geometric probability (i.e. a target).

Example:

If you choose a point in the square, what is the probability that it is not in the circle?



Instructional Strategies: See **Grade 7.SP.5**

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Resources/Tools:

[7.SP.7a How Many Buttons?]

[Is it Fair?", Georgia Department of Education.]

Students play the game "Is It Fair?" and record their information using probability to determine whether they feel the game is fair or not. Predictions are made before the game begins. Based on their trials, students determine all outcomes, create tree diagrams and determine the theoretical chance of winning for each player.

Common Misconceptions:

Students often expect the theoretical and experimental probabilities of the same data to match.

By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities.

Note examples in simulations when some possibilities are not shown.

Domain: Statistics and Probability (SP)

Cluster: *Investigate chance processes and develop, use, and evaluate probability.*

Standard: Grade 7.SP.8

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
- **c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See **Grade 7.SP.5**

Explanations and Examples:

Students use tree diagrams, frequency tables, and organized lists, and simulations to determine the probability of compound events.

Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

Examples:

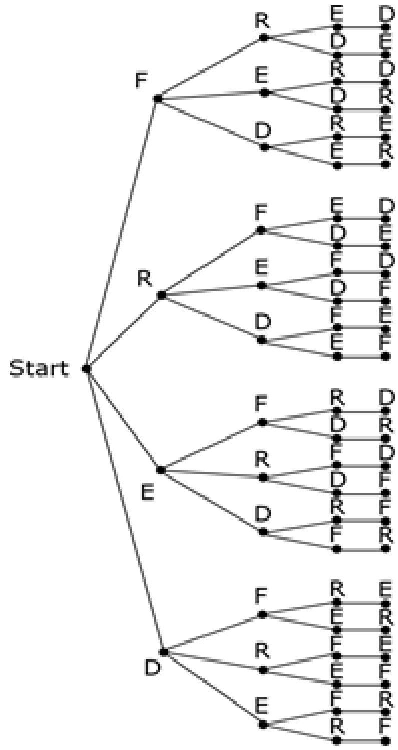
Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another.

What is the sample space for this situation? Explain how you determined the sample space and how you will use it to find the probability of drawing one blue marble followed by another blue marble.

Show all possible arrangements of the letters in the word FRED using a tree diagram.

If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order?

What is the probability that your "word" will have an F as the first letter?



Instructional Strategies:
Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome.

Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability. Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

Resources/Tools:

7.SP Waiting Times
7.SP Rolling Twice
7.SP Red, Green, or Blue?
7.SP Sitting across from Each Other
7.SP – Tetrahedral Dice
7.SP Sitting across from Each Other
7.SP – Tetrahedral Dice

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APPENDIX

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with ”fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every (a) there exists $(-a)$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

*TABLE 4. The Properties of Equality**

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$ then $b=a$
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$
Addition property of equality	If $a=b$ then $a+c=b+c$
Subtraction property of equality	If $a=b$ then $a-c=b-c$
Multiplication property of equality	If $a=b$ then $a\times c=b\times c$
Division property of equality	If $a=b$ and $c\neq 0$ then $a\div c=b\div c$
Substitution property of equality	If $a=b$ then *b* may be substituted for *a* in any expression containing *a*.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a<b, a=b, a>b$.
If $a>b$ and $b>c$ then $a>c$
If $a>b$ then $b<a$
If $a>b$ then $-a<-b$
If $a>b$ then $a\pm c>b\pm c$
If $a>b$ and $c>0$ then $a\times c>b\times c$
If $a>b$ and $c<0$ then $a\times c<b\times c$
If $a>b$ and $c>0$ then $a\div c>b\div c$
If $a>b$ and $c<0$ then $a\div c<b\div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Table 6 Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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